

Upscaling Toughness in Hydraulic Fractures: Insights from Periodic Variations

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We discuss whether and how an averaging-based approach to material toughness can be confidently utilized. Usually, various upscaling procedures are applied to achieve the goal. Recently, we have proposed a new averaging-based approach dependent on the material and process-dependent parameters. The respective measures come from temporal averaging (in contrast to the spatial one). They require a knowledge of the instantaneous crack tip velocity during each specific process. The temporal average approach is general in its nature, and can be used in the analysis of any stable fracture propagation process.

Numerous simulations have been performed to verify the measure proposed. We used hydraulic fracture as it always produces a stable crack propagation. We utilize our extremely accurate and effective in-house built time-space adaptive solver, which can obtain solutions for all classic HF models (PKN, KGD, Radial) with arbitrary fluid rheology, leak off and pumping regime. The solver uses the crack opening and the fluid velocity as the basic unknowns in contrast to the conventional crack opening and fluid pressure pair [1]. We analyse the KGD and Radial HF models in an elastic homogeneous material characterised by periodic toughness distributions [2-4]. In particular, we show how local energy redistribution affects the process, resulting in local (in time and space) changes in the propagation regime. For example, even if both the maximum and minimum values of the toughness distribution correspond solely to the high toughness regime (under a given fluid rate), local regions exhibiting viscosity-dominated behaviour are apparent. Another interesting feature of the proposed measures: even though the toughness and energy release rate fracture criteria are equivalent in the problem under consideration (homogeneous elastic material), temporal averaging based on the energy argument appears more accurate. Finally, we show an interesting effect of the fluid reversal within the fracture for a small-time fraction and question the quasi-static approach commonly utilised in modelling propagation of HF fracture in inhomogeneous material.

- [1] Wrobel, M. Mishuris, G. (2015) Hydraulic fracture revisited: Particle velocity-based simulation, *Int. Journal Engineering Sciences*, 94, 23-58, [10.1016/j.ijengsci.2015.04.003](https://doi.org/10.1016/j.ijengsci.2015.04.003)
- [2] Peck, D., Da Fies, G., Dutko, M., Mishuris, G. (2022). A temporal averaging-based approach to toughness homogenisation in heterogeneous material. *Mathematics and Mechanics of Solids*, <https://doi.org/10.1177/108128652211117>.
- [3] G. Da Fies, M. Dutko, D. Peck (2023) Averaging-based approach to toughness homogenisation for radial hydraulic fracture, <https://arxiv.org/abs/2211.03114>
- [4] D. Peck, G. Da Fies, I. Virshylo, G. Mishuris (2025) Peculiarities of hydraulic fracture propagation in media with heterogeneous toughness: the energy balance, elastic battery and fluid backflow, <https://arxiv.org/abs/2410.20139>